

# **Material properties and theory for structural analysis (1)**

**(The basic theory of property for structural analysis)**

R&D Center

Though a technique might see continuous development, structural analysis can't fully actual conditions. Even loads and boundary conditions never perfectly match the actual test conditions. As such, engineers mention that the results of the analysis have to be used as a reference and include a variety of assumptions.

Can we do it roughly, then? It's no different from giving up their capacity as an engineer. Then how can we decrease the gap between actual status and analysis data? The procedure of solving these worries could be the development process by all engineers from the person who first introduced the CAE.

In addition, the inherent method to correlate structural analysis and actual test results of injection parts is to determine how to input the properties of materials. Accordingly, regarding the material properties for structural analysis, we would like to take a two-pronged approach as detailed below:

- ▶ The basic theory of solid & material mechanics for understanding structural analysis
- ▶ The process of input properties for improving precision of the analysis

Naturally, save for experts, most are not interested in solid mechanics, so instead of arranging deeper theories or formulae, our strategy consists of essential factors for understanding structural analysis.

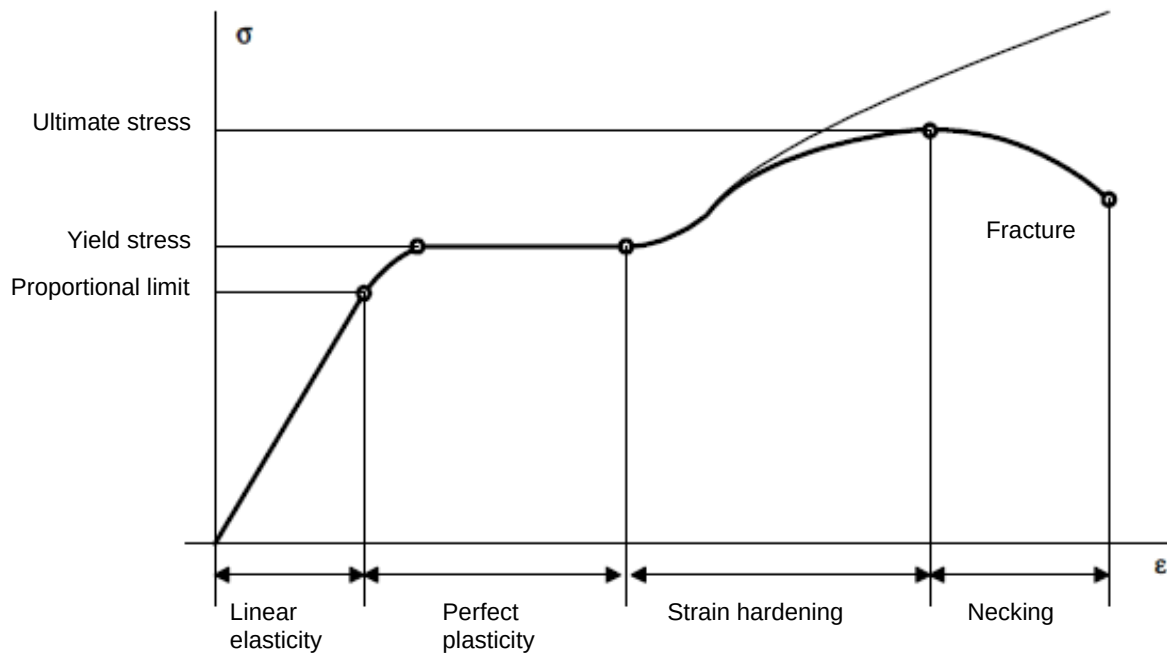
## ◎ The basic theory of property for structural analysis

### (1) Basic understanding

In general, for analyzing static loads, essential properties are modulus (Young's Modulus or Elastic Modulus) and Poisson's Ratio. The word "general" here means "elastic behavior is dominant". Because metallic material shows elastic-like behavior before breakage, and after yield point area, the behavior of stress and strain carries no great meaning any longer.

However, most resins exhibit vague boundaries between elastic and plastic areas, and, in many cases, also cannot be ignored in the behavior of the plastic area save for fatigue loads or creep loads. In other words, the structural analysis of a common resin using only with modulus and Poisson's Ratio is not accurate, and as such present the need for something that represents material behavior.

The strain-stress curve is necessary at this point, and the analysis including the general behavior of a resin, such as the right side may extend the analysis range to the plastic area including elastic ones.



[Fig.1] The Stress-Strain curve

## (2) Stress – Strain curve

Stress is the internal resistance force of materials regarding the load, the value, and exactly the load per unit area. Therefore, if the load is large, the stress is bigger than the resistance so that it will break, but does not break with less stress because of its internal resistance force.

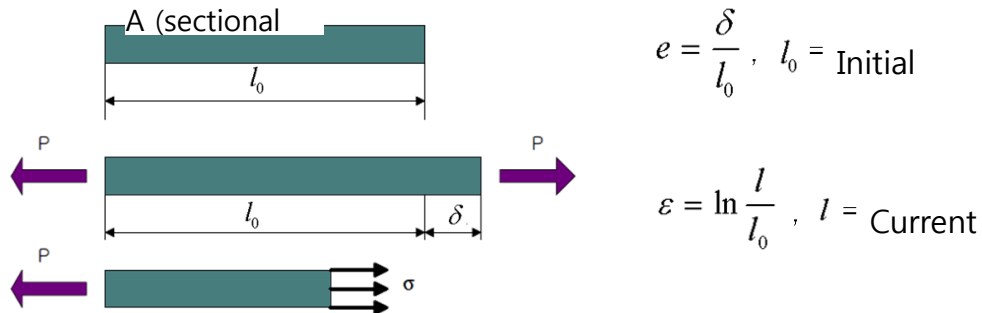
Strain refers to a change in length-per-unit length under tensile or compressive load; normally during tensile or compressive tests, stress and strain are recorded as a pair.

The graph which demonstrates this pair is called the Stress-Strain curve (after the SS curve) as mentioned in the previous section.

In general, when using a UTM (Universal Test Machine) in a tensile test of the specimen, the load (F) - displacement (D) data can be obtained as a raw data type, which can also output the Engineering Stress - Engineering Strain graph automatically.

One thing that have to understand is the difference between Engineering Stress and True Stress. In case of UTM, it calculates the Engineering Stress-strain graph automatically, however actual analysis is used in the True Stress-True Strain graph. (Recent UTM can calculate the True Stress-strain.) Precisely, the True Stress-Plastic Strain graph is used, but will be converted into one more from the True Stress-strain and this paper will explain the details further in.

### (3) Engineering Strain & True Strain



Engineering Strain( $e$ ) is the ratio of the length variation from the length before deformation (gauge length of original specimens), True Strain( $\varepsilon$ ), also called Logarithmic Strain or Natural Strain is the ratio of the length variation based on current length.

In plain words, while Engineering Strain is the value that variation of gauge length is measured by each point divided into initial gauge length, and True Strain is the value that variation of gauge length is measured by each point divided into gauge length changed immediately before. In other words, Engineering Strain does not take into account the length of the altered specimens, True Strain considers the length of the specimens to be changed every time and so can be considered the real strain.

A normal tensile test can easily calculate Engineering Strain as it records the specimen's initial gauge length before measuring moved distance. However, True Strain is calculated based on the gauge length of altered specimens at the point of every measuring moment, re-calculating its real variation amount, so in many cases, it cannot directly display the measured value in the machine. Therefore, in most cases, UTM is to use the readily measurable Engineering Strain.

Accordingly, the definition of True Strain

$$d\varepsilon = \frac{dl}{l} \rightarrow \varepsilon = \int d\varepsilon = \int_{l_0}^l \frac{dl}{l}$$

is obtained, this can express True Strain as Engineering Strain.

(True Strain :  $\varepsilon$ , Engineering Strain :  $e$ )

$$\varepsilon = \ln \frac{l}{l_0} = \ln \left[ \frac{l - l_0 + l_0}{l_0} \right] = \ln(1 + e) \quad \text{or} \quad e = \exp(\varepsilon) - 1$$

Using this relation, Engineering Strain values from UTM can be converted to True Strain.

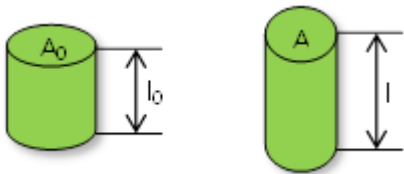
#### (4) Engineering Stress & True Stress

The way to distinguish Engineering Stress and True Stress is similar to the definition of the strain do. That is, “engineering” is based on the initiated stage, “true” from current. While Engineering Stress is calculated based on the initial cross-sectional area of the specimen, True Stress is based on the actual varying cross-section area of the tensile test.

Looking closely at a tensile test on the specimen, in accordance with the test progression, the cross sectional area of the specimen in the gauge length area slowly decreases. Therefore, as with Engineering Stress, if the load divides into the initial cross-sectional area of the specimen, it cannot be said that the measurement is real stress because the testers do not take into account the varying cross-sectional area.

So, by converting Engineering Stress into recalculated stress based on the cross-sectional area which changes every moment for analyzing, this stress is called “True Stress”. However, the assumption that volume within the gauge length is always constant, - called “isochoric”- is used when converting Engineering Stress into True Stress.

As shown in the figure below, the assumption that volume of the initial gauge length is equal to the volume in the changed gauge length during tensile test, " $A_0 l_0 = A l$ " can be referred to, which formulae also can be modified as like below. (Engineering Strain:  $e$ )



$$\frac{A_0}{A} = \frac{l}{l_0} = \frac{l - l_0 + l_0}{l_0} = \frac{\delta}{l_0} + 1 = e + 1$$

$$\therefore \frac{A_0}{A} = e + 1$$

Writing as like below followed by the definition of Engineering Stress and True Stress,

(Engineering Stress :  $S$ , True Stress :  $\sigma$ , Engineering Strain :  $e$ )

$$S = \frac{P}{A_0}, \quad A_0 = \text{Initial sectional area}$$

$$\sigma = \frac{P}{A}, \quad A = \text{Current sectional area}$$

$$= \frac{P}{A_0} \times \frac{A_0}{A} = S(e + 1)$$

That is, True Stress can be converted to the formulae of Engineering Stress and Strain like above.

Synthesizing True Strain and Stress formulae explained above shows like below,

$$\varepsilon = \ln(1 + e), \quad \sigma$$

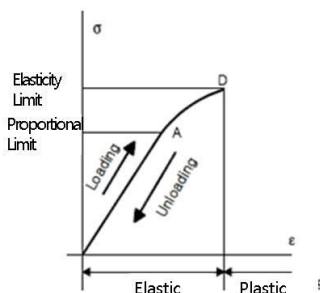
now measured Engineering Strain and Stress using this two equations, it is possible to easily transform into True Stress and Strain.

## (5) Elastic Modulus

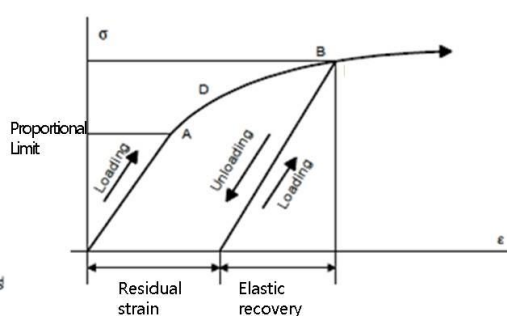
In general, material is elastically deformed under a small load and the stress is in proportion to the amount of deformation; this is called "Hook's Law", represented by the equation : stress( $\sigma$ ) / strain ( $\varepsilon$ ) = modulus of elasticity (E).

In general, Young's Modulus(E) and the transverse modulus of elasticity is represented by (G) normally, though, it means Young's Modulus and, if it is determined that the behavior of the material in a very small linear elastic region (see Figure 1), only Modulus (E) and Poisson's Ratio( $\nu$ ) can be enough to conduct a static load structural analysis.

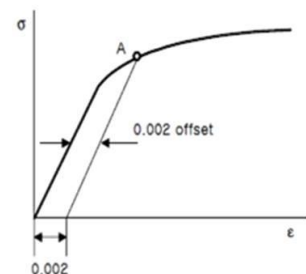
In the case of metallic materials, as elastic limit and proportional limit are almost similar, the end of the linear elastic region can be defined as yield stress without wide errors, and it can be analyzed using only the modulus of elasticity.



[Fig. 2] Elasticity limit



[Fig. 3] Proportion limit



[Fig. 4] Offset Yield Stress

(Proportional limit: the maximum area of linear stress-strain relation,

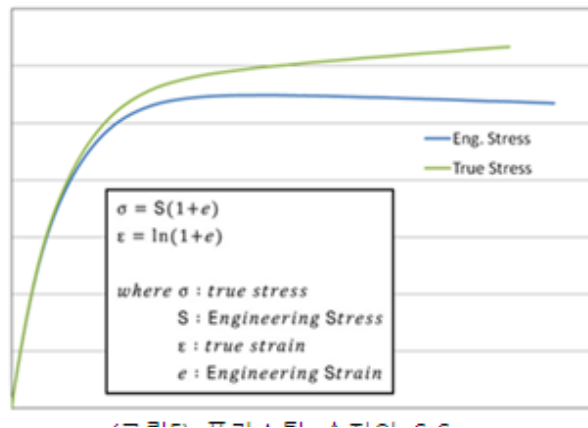
the elastic limit: the maximum area when removing the load back to the initial size)

However, most resin has a very short linear elastic region or is difficult to grasp, therefore it becomes impossible to find the yield point by using the definition of yield stress (0.2% offset) of common metal.

Therefore, structural analysis using a resin has to use a method for inputting the SS curve directly, otherwise it can't express the behavior of materials in the linear elastic limit only with elastic modulus.

Of course, even with a very short section, there exists a need to define the elastic region and input modulus.

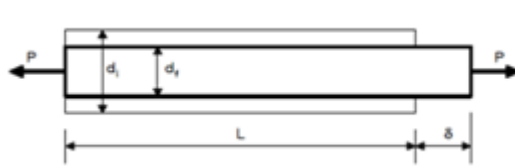
As with (Fig. 5), resin indicates that the elastic section and the yield point is ambiguous, and so elastic modulus and yield stress can be determined according to the ISO standards for the SS curve of several forms. (Please refer to details further in this paper.)



[Fig. 5] S-S curve of general plastics

## (6) Poisson's Ratio

When applying a load in a certain axis direction and consequently deforming in that direction, the ratio of deformation on the opposite side is called Poisson's Ratio. Easily put, when pulling out the tensile specimen, the proportion of the reduced amount of deformation in the vertical direction from load with respect to the increased deformation amount in the load direction. Basically, it can be used at the elastic region because of the assumption of Hooke's Law, theoretically having a value less than 0.5 greater than zero.



$$\epsilon_{lat} = \epsilon_t = \frac{d_f - d_i}{d_i}, \quad \epsilon_{long} = \epsilon_a = \frac{\delta}{L}$$

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}} = -\frac{\epsilon_t}{\epsilon_a} \quad 0 \leq \nu \leq 0.5$$

Common carbon steel's Poisson's Ratio is 0.28 to 0.3, and rubber is almost close to 0.5. Our flagship product, F20-03 is about 0.39 or so. The structural analysis must be entered in the required Poisson's Ratio, most resin is often difficult to analyze because its Poisson's Ratio is not well known.

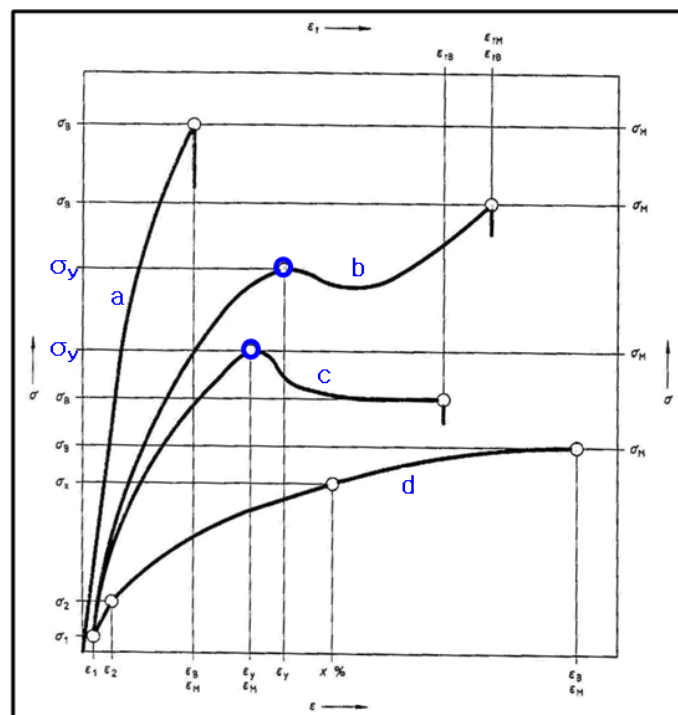
In this case, using a similar resin's Poisson's Ratio from material suppliers, experientially, with a low degree of error ( $\pm 0.03$  degree) can't affect analysis errors for analyzing static load, so there is no need to worry about it. However, the error of frequency per mode occurs during vibration analysis.

## (7) Yield Stress

As described above, yield stress is the point at which permanent deformation is drastically increased when an external force is increased beyond the elastic limit. As for metallic material, the maximum stress of the elastic limit is the same as yield stress.

If you remove the load in the tensile test using general specimens, the maximum stress value returning to the initial point of the SS curve becomes yield stress; even it returns, it has an error with the permanent deformation, Mechanics of Materials recommend using the offset yield stress that allows 0.05 to 0.3 percent of permanent strain. (See Fig. 4)

However, the resin is used to define yield stress in accordance with ISO regulations as these theories cannot be used as-is because of a tendency to show a markedly different SS curve than that of metal.

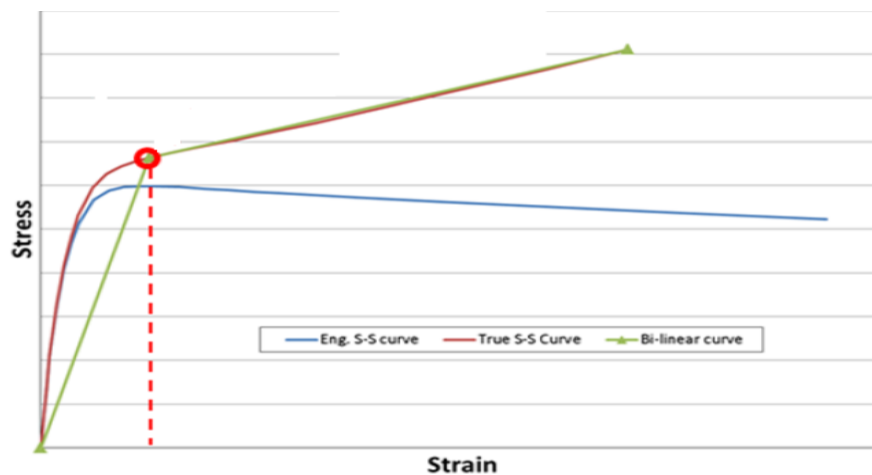


[Fig. 6] S-S curve types of plastics

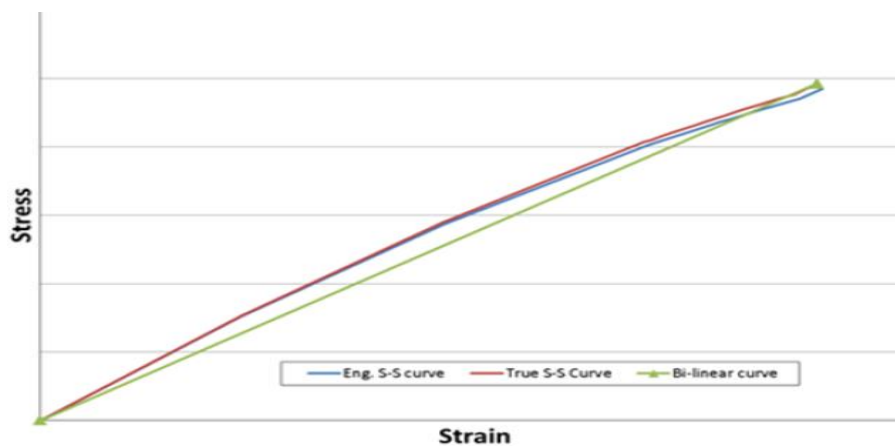
- If the forms of the graph are a and d, yield stress is not represented.
- If the forms of the graph are b and c,  $\sigma_y$  is the yield stress, and may be less than the maximum stress.
- If the form of the graph is d, originally there is no yield stress but could be defined stress in the particular strain point "X" established between the parties as yield stress.

- In addition, the elastic modulus in accordance with ISO regulations is defined by a gradient between 0.0005 to 0.0025 range of strain, and this provision is the criterion of engineering stress and strain. (Fig. 7 S-S curve of KEPITAL F20-03)

Therefore, when the stress to the linear elastic region is yield stress, such as metal, it cannot be used because the error is too large to be a yield stress at the point of a crossing between the linear elastic modulus and SS curve according to ISO standards. Below is the SS curve of typical KEPITAL F20-03 and MAXIMID 7550GF among KEP company's products corresponding to lines c and d.



[Fig. 7] S-S curve of KEPITAL F20-03



[Fig. 8] S-S curve of MAXIMID 7550GF

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